Cosmology and Gravitostatics

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ABSTRACT

In gravitostatics, the minimally generalized Newtonian theory, the simplest cosmological model implies a cosmological redshift with acceleration parameter -1. If the gravitational potential satisfies a wave equation, the total mass density (visible plus dark) is one third of the critical density, in good agreement with observation. There is no horizon problem and no need to invoke dark energy.

Subject headings: gravitation — cosmology: theory — galaxies: high redshift — dark matter — early universe

1. Introduction
The empirical evidence that might distinguish between different, relativistic theories of gravity is very sparse. Any acceptable theory must of course be compatible with the Newtonian theory. It used to be thought that the small deviations from the Newtonian predictions, the so-called 'classical tests of relativistic gravity', can be used to discriminate between relativistic theories. However, a slightly modified Newtonian theory, called *gravitostatics,* accounts for these deviations. Any relativistic theory must satisfy the classical tests, but this shows only that it is not incompatible with gravitostatics. (One must emphasize that gravitostatics is not a relativistic theory, although it reduces to special relativity in the limit when all gravitational fields vanish.) To choose between different compatible theories, other experiments are needed.

In addition to the classical tests, there are other, small, 'postnewtonian'deviations from the Newtonian predictions, but they yield only weak constraints on possible theories. The most promising testing ground for relativistic theories is the study of gravitational radiation. Although experiments are very difficult, there is one well-established, observational result: the rate of energy loss by gravitational radiation from a binary pulsar. This agrees with the prediction of the Einstein theory, and is the only good piece of evidence for it at present. The Einstein theory is widely accepted for aesthetic reasons, and because of the misapprehension that the classical tests support it uniquely, as discussed in the last paragraph.

Cosmology is another field for the testing of gravitational theories. The Einstein theory had difficulties here. There was the 'horizon problem': spatially separated regions are causally disconnected at early times. There was also the 'flatness problem': unless the initial conditions are chosen with extreme precision, the universe does not unfold as it should. Both difficulties were ingeniously overcome by the theory of inflation. It is still unclear whether this produces exactly the correct conditions for the subsequent expansion of the universe; even if it does, many feel that it is too contrived and that it destroys the aesthetic appeal of the theory. Recently, measurements of very distant objects have shown that the Einstein theory is admissible only if the universe is filled with an otherwise unobservable 'dark energy'. The invention of such entities is a mark of theories under strain: one thinks of phlogiston and the ether.

In this uncertain situation, it is illuminating to see what gravitostatics says about cosmology. This requires another small generalization of the theory. the Poisson equation, inherited from the Newtonian theory, must be modified by the addition of a term involving time derivatives. In this paper, the cosmological predictions of the modified theory are shown to be free from the problems of the conventional theory. The way is now open for the investigation of properly relativistic theories with the same qualities.

The rationale for this paper is that, since the empirical evidence for the Einstein theory is not strong, it is worthwhile to investigate other theories. This will reveal a wider range of possibilities and will help to distinguish real features of the world from merely theoretical constructs. It is suggested that inflation and dark energy may be examples of such constructs. The opposing view, more widely held, is that there is no point in developing other theories so long as observations do not clearly contradict the Einstein theory.

Which opinion one adopts is a matter of taste or temperament. It requires cool assessment, not religious fervor.

2. Gravitostatics

It should be well known that the Newtonian theory of gravitation can be generalized to be compatible with special relativity in the limit of vanishing gravitational fields, and to account for the gravitational redshift and the other classical tests of relativistic gravity. The generalized theory, *gravitostatics,* is non-speculative, in that it makes no new hypotheses but simply takes the old ones more seriously. For example, one finds that the assumption that the gravitational potential is arbitrary to the extent of an additive constant implies that the rate of a clock depends exponentially on the gravitational potential — and that the units of length and mass have a similar dependence. A full account of all this can be found in (Rastall 1991); a shorter, more accessible source may be (Rastall 1975). The next paragraphs try to summarize enough of the theory to make the rest of the paper comprehensible.

In gravitostatics, there are preferred coordinate systems, called *Newtonian frames*, which are similar to the Galilean frames of Newtonian theory or the inertial charts of special relativity. As in Newtonian theory, the gravitational field is described by a function Φ, the *gravitational potential.* The measured values of universal constants such as the speed of light c the Newtonian gravitational constant G, and Planck's constant h are assumed to be independent of Φ. In a static gravitational potential (that is, when Φ may depend on the space coordinates $\mathbf{x} = (x^1, x^2, x^3)$ of a Newtonian frame but not on the time coordinate t) the time interval measured by a fixed standard clock is not in general equal to the change in the time coordinate t. Similarly, the distance as measured with a standard measuring rod between the spatial points with coordinates $\mathbf{x}$ and $\mathbf{y}$ is not in general $|\mathbf{x} - \mathbf{y}| = [(x^m - y^m)(x^m - y^m)]^{1/2}$ (the summation convention is assumed unless stated otherwise: sum the repeated index m over its range $(1, 2, 3)$). The spatial geometry as measured by standard measuring rods is consequently not Euclidean. Directly measured times and lengths will be termed *empirical*; those defined in terms of the space and time coordinates of a Newtonian chart will be termed *Newtonian.*

The energy of an object at any point can be measured in terms of the energy of a reference object at a nearby point (the reference object might be a standard slug of metal or a specified atom in its ground state, for example). This is called the *empirical energy.* From the laws of mechanics, which are a trivial generalization of the special relativistic laws, it follows that there is an energy function which is constant along the path of a particle in a static gravitational potential. This is called the *Newtonian energy*, and is in general different from the empirical energy.

Empirical quantities may be denoted by a subscript E and Newtonian quantities by a subscript N. For local quantities — those which are defined in a region where Φ is almost constant — one proves that the ratio of a Newtonian quantity to the corresponding empirical quantity is an exponential function of Φ (this follows from the assumption that Φ is arbitrary to the extent of an additive constant). If t_N, ℓ_N, e_N are a time interval, length, and energy measured in Newtonian units, and t_E, ℓ_E, e_E are the corresponding quantities in empirical units, one has

$$t_N = t_E \exp[-\tau(\phi - \phi_0)], \qquad \ell_N = \ell_E \exp[-\lambda(\phi - \phi_0)], \tag{1.1}$$

$$e_N = e_E \exp[-\eta(\phi - \phi_0)],$$

where τ, λ, η, and ϕ_0 are constants, $\phi = \Phi/{c_E}^2$, and c_E is the speed of light in empirical units. One calls ϕ the *dimensionless potential.* Newtonian and empirical values are equal when $\phi = \phi_0$; a Newtonian frame for which this is true is called a ϕ_0 *frame.*

The magnitude of a dimensionless quantity is independent of the units of measurement. It follows that, when measured in Newtonian units, the magnitudes of two quantities with the same dimensions must have the same dependence on ϕ (because their ratio is dimensionless). Since the dimensions of a mass m are $[m] = [e\ell^{-2}t^2]$, eq.(1.1) gives

$$m_N = m_E \exp[-\mu(\phi - \phi_0)], \tag{1.2}$$

where $\mu = \eta - 2\lambda + 2\tau$. To avoid an excess of subscripts, one makes the convention that universal constants without a subscript E or N are to be interpreted as empirical quantities (the speed of light c is to be interpreted as c_E, Planck's constant h as h_E, etc.). Anything else written without a subscript E or N (i.e. anything but a universal constant) is to be interpreted as a Newtonian quantity, unless stated otherwise (the velocity $\mathbf{V}$ of a particle is to be interpreted as $\mathbf{V}_N$, etc.).

We have now set up the conceptual framework of gravitostatics. To go further, we must learn how to calculate the motion of a particle in a gravitational field. This would seem to require some additional, arbitrary hypothesis. However, the Lagrangian method allows us to proceed in a non-arbitrary, or at least very natural way.

The Lagrangian of a free particle in special relativity is $L = -mc^2/\gamma$, where m is the constant and invariant proper mass, $\gamma = (1 - V^2c^{-2})^{-1/2}$, and $V = |\mathbf{V}|$ is the speed. The guiding principle of gravitostatics is to adopt the laws of special relativity and Newtonian gravity with minimal changes. One therefore assumes that the Lagrangian of a particle subject only to a gravitational field is the same as that for a free particle — except that everything is to be measured in Newtonian units. With the previous conventions, one has

$$L = -m{c_N}^2/\gamma, \qquad \gamma = (1 - V^2{c_N}^{-2})^{-1/2}, \tag{1.3}$$

where m is the proper mass in Newtonian units. As in Newtonian mechanics, it is assumed that m_E, the proper mass in empirical units, is constant along the path. (It would of course be more correct to write the Lagrange function as $L(\mathbf{Z}, \mathbf{V}, \cdot)$, where $\mathbf{Z}$ is the position of the particle, and its value at time t as $L(\mathbf{Z}(t), \mathbf{V}(t), t)$.) From (1.1), one gets

$$c_N = c\exp[(\tau - \lambda)\psi], \qquad m{c_N}^2 = m_E c^2 \exp(-\eta\psi), \tag{1.4}$$

where $\psi = \phi - \phi_0$. If ψ and V are small, the Lagrangian must be equivalent to the Lagrangian $m_E V^2/2 - m_E c^2\phi$ of Newtonian gravitation, which implies that $\eta = -1$.

The r component of the momentum of the particle is $p_r = \partial L/\partial V^r = m\gamma V^r$ and the Euler-Lagrange equation is $dp_r/dt = \partial L/\partial Z^r$. Since $V^r p_r - L = m\gamma(V^2 - {c_N}^2\gamma^{-2}) = m\gamma {c_N}^2$, one has

$$(d/dt)(m\gamma {c_N}^2) = -\partial_t L, \tag{1.5}$$

where $\partial_t L$ denotes the partial derivative with respect to the last variable in $L = L(\mathbf{Z}, \mathbf{V}, \cdot)$. If ϕ is static, then $\partial_t L = 0$, and $E = m\gamma c_N{}^2$ is constant along the path of the particle. It is identified with the *energy* of the particle in Newtonian units. The Newtonian unit of energy can thereby be defined everywhere in the Newtonian frame. One may choose the Newtonian unit to be the same as the empirical unit in a region where the potential is ϕ_0. (If ϕ is quasistatic, as will later be allowed, then (1.5) gives the change in E along the path.)

Rapidly moving objects are permissible in gravitostatics, provided that they do not contribute significantly to the gravitational field. One can introduce a classical model of a photon as the limiting case of a particle whose mass tends to zero and whose speed tends to the speed of light in such a manner that its energy $E = m\gamma {c_N}^2$ remains bounded. The momentum of the photon is parallel to its velocity and has magnitude E/c_N. In a static potential, the Newtonian energy E of the photon is constant along its path.

A photon of Newtonian energy E is associated with an electromagnetic wave of Newtonian frequency ν. Planck's law states that $E_E/\nu_E = h$, where h is Planck's constant, or equivalently that $E/\nu = h_N$. In a static potential, the Newtonian energy E of the photon is constant along its path, and the Newtonian frequency ν of the associated wave is constant (otherwise, the number of wavelengths between two spatial points is not constant in time). Hence E/ν is constant along the path. At the potential ϕ_0, where Newtonian units are the same as empirical, one has $E/\nu = h$ and it follows that, at any potential, $E/\nu = h_N = h$: the value of Planck's constant is the same in Newtonian and empirical units. Since $h_N = h\exp[-(\eta + \tau)\psi]$, from (1.1), we have $\tau = -\eta = 1$.

In the Newtonian theory, the gravitational potential Φ obeys Poisson's equation $\partial_m \partial_m \Phi = 4\pi G\rho$, where G is the gravitational constant and ρ is the mass density of the sources. In gravitostatics, the only difference is that one uses the energy density ϵ of the sources:

$$\partial_m \partial_m \phi = 4\pi k_N \epsilon, \tag{1.6}$$

where $\phi = \Phi c^{-2}$ and $k_N = G_N {c_N}^{-4}$, or $k = Gc^{-4}$(note that Φ is measured in empirical units).

In Newtonian theory, the active gravitational mass of an object, which determines the strength of the gravitational field which it produces at large distances, is the same as the passive gravitational mass (the weight) and the inertial mass. In gravitostatics, similarly, the active gravitational energy is assumed to be equal to the total energy of the sources. A short calculation shows that this implies $\lambda = \eta$. All three constants in (1.1) are now determined:

$$\tau = -\lambda = -\eta = 1, \tag{1.7}$$

and eqs.(1.4) and (1.3) become

$$c_N = ce^{2\psi}, \qquad m{c_N}^2 = m_E c^2 e^{\psi}, \qquad L = -m_E c^2 e^{\psi}(1 - V^2 c^{-2} e^{-4\psi})^{1/2}. \tag{1.8}$$

Using the values (1.7), one calculates the gravitational redshift, the bending of light by the Sun (and the related radar time-delay), and the anomalous perihelion shift of Mercury. These are often called the *classical tests of relativistic gravity*. In fact, they are predicted by gravitostatics, which is only a slight generalization of the Newtonian theory.

3. The cosmological redshift

Like Newtonian theory, gravitostatics can be applied when the gravitational field is slowly varying in time. The simplest cosmological models are those in which the gravitational potential is independent of the space coordinates and is a function only of the time coordinate t of a Newtonian frame. We take the special case when the potential is a linear function of the empirical time T:

$$\psi = \phi - \phi_0 = -HT, \tag{2.1}$$

where ϕ_0 and $H > 0$ are constants and T is a function of t only. Since T at any spatial point can be regarded as time measured by a standard clock fixed at that point, one has $dT/dt = e^{\psi} = e^{-HT}$ from (1.1) with $\tau = 1$. Integrating and choosing $T = 0$ when $t = 0$, one gets $e^{-\psi} = Ht + 1$. As $t \longrightarrow \infty, \psi \longrightarrow -\infty$ and $T \longrightarrow \infty$; as $t \longrightarrow -1/H, \psi \longrightarrow \infty$ and $T \longrightarrow -\infty$. In this model the 'horizon problem' of cosmology does not arise: since $c_N = ce^{2\psi}$, signals can, in principle, be sent to any spatial point $\mathbf{x}$ at a time t from any other spatial point $\mathbf{x}'$ at a time t', provided that t' is sufficiently close to the value $-1/$H.

To define the redshift, we suppose that light emitted by an atom at rest at the spatial point 1 in a Newtonian chart has frequency ν_E, measured in empirical units. It travels to the spatial point 2, where its frequency is measured to be ν_{2E}. This is compared with the frequency ν_E of the light emitted from an identical atom at rest at the spatial point 2. The redshift z (of the light from 1 as measured at 2) is defined by

$$z = (\nu_E - \nu_{2E})/\nu_{2E} = (E_E - E_{2E})/E_{2E}, \tag{2.2}$$

where the frequency ν_E of the light and the energy E_E of the corresponding photon are related by $\nu_E = E_E/h$, and similarly $\nu_{2E} = E_{2E}/h$.

It is easiest to calculate the redshift by again considering the motion of a classical 'photon'. The Newtonian energy of such a photon changes according to (1.5). From (1.8) one has $\partial L/\partial\psi = -E(1 + V^2 {c_N}^{-2})$, and putting $V = c_N$ gives

$$dE/dt = 2E\partial_t \psi. \tag{2.3}$$

As before, a photon of empirical energy E_E is emitted by an atom at rest at the spatial point 1 at the instant t_1 in a Newtonian chart. The Newtonian energy of the photon is $E_1 = E_E e^{\psi_1}$, from eqs.(1.1) and (1.7), where $\psi_1 = \phi_1 - \phi_0$ and ϕ_0 is a constant. The photon travels to the point 2, where its Newtonian energy is E_2. From (3.3), $\ln(E_2/E_1) = \int 2\partial_t \psi(\mathbf{Z}(t), t)dt$, where the integral is from $t_1$to t_2. The photon's empirical energy at point 2 is

$$E_{2E} = E_2 e^{-\psi_2} = E_E \exp[\psi_1 - \psi_2 + \int_{t_1}^{t_2} 2\partial_t \psi(\mathbf{Z}(t), t)dt]. \tag{2.4}$$

From (3.2), the redshift is

$$z = (E_E/E_{2E}) - 1 = \exp[\psi_2 - \psi_1 - \int_{t_1}^{t_2} 2\partial_t \psi(\mathbf{Z}(t), t)dt] - 1. \tag{2.5}$$

In the special case of a static potential, one has $\partial_t \psi = 0$ and the redshift is $z = \exp(\psi_2 - \psi_1) - 1 \approx \psi_2 - \psi_1$ if $|\psi_2 - \psi_1| \ll 1$. Note that the redshift is positive if the light goes from a lower to a higher potential. If $\partial_m \psi = 0$ everywhere, so that ψ is a function of t only, eq.(2.5) becomes

$$z = \exp(\psi_1 - \psi_2) - 1. \tag{2.6}$$

Here the redshift is positive if the light goes from a higher to a lower potential.

In the special case when $\psi = -HT$, eq(2.1), the redshift is

$$z = \exp[H(T_2 - T_1)] - 1 = H(T_2 - T_1) + (1/2)H^2(T_2 - T_1)^2. \tag{2.7}$$

The last equation is valid if $H(T_2 - T_1)$ is small. The term in H^2 corresponds to a deceleration parameter $q = -1$, in the conventional terminology (see eq. (6.2) or eq.(29.15) of (Misner, Thorne, & Wheeler 1973)). Eq.(2.7) implies that the universe is 'accelerating' — it expands more rapidly at later times. This agrees with recent measurements on the redshifts of distant supernovae and galaxies (Knop, et al. 2003), (Tonry, et al. 2003), (Daly & Djorgovski 2003).

4. A field equation and dark mass

The field equation (1.6) is the same as the classical Poisson equation, except that the mass density of the sources is replaced by their energy density. An apparent defect of the equation is that the energy density of the gravitational field does not appear in the source term. It is however easy to remedy this: instead of choosing the field variable to be ϕ, or $\psi = \phi - \phi_0$, one takes it to be $e^{\psi/2}$. One then has $2e^{-\psi/2}\partial_m\partial_m e^{\psi/2} = \partial_m\partial_m\psi + (1/2)\partial_m\psi\partial_m\psi$ and (1.6) becomes

$$2e^{-\psi/2}\partial_m\partial_m e^{\psi/2} = 4\pi k_N(\epsilon + \epsilon_G), \tag{4.1}$$

where $\epsilon_G = (8\pi k)^{-1}\partial_m\psi\partial_m\psi$ is the energy density of the gravitational field. Since the dimensions of $k = Gc^{-4}$ are length divided by energy, we have $k_N = k$, from (1.1) with $\eta = \lambda = -1$. Note that ϵ_G is never negative — unlike the energy density in Newtonian gravitation, which is $-\epsilon_G$.

Eq. (4.1) is completely equivalent to (1.6). It is the equation satisfied by a static potential ψ. If the potential is not static, then the field equation must involve time derivatives if it is to determine the time evolution of the potential. Here a speculative element enters the theory. We guess, as the most obvious possibility, an equation similar to the wave equation:

$$2e^{-\psi/2}[\partial_m\partial_m e^{\psi/2} - c_N{}^{-1}\partial_t(c_N{}^{-1}\partial_t e^{\psi/2})] = 4\pi k(\epsilon + \epsilon_G). \tag{4.2}$$

(The c_N are inserted to ensure that the equation is invariant under change of ϕ_0 chart.)

If we assume that $\psi(t) = -HT$, as in (3.1), and use $c_N = ce^{2\psi}$ and $dT/dt = e^{\psi}$, we have

$$2e^{-\psi/2}{c_N}^{-1}\partial_t({c_N}^{-1}\partial_t e^{\psi/2}) = -(H^2/2c^2)e^{-2\psi}.$$

Defining the *critical density* ϵ_c by $\epsilon_{cE} = 3H^2/8\pi kc^2$, and $\epsilon_c = \epsilon_{cE}e^{-2\psi}$, we find that (4.2) becomes

$$2e^{-\psi/2}\partial_m\partial_m e^{\psi/2} = 4\pi k(\epsilon + \epsilon_G - \epsilon_c/3). \tag{4.3}$$

Since the space derivatives vanish in this cosmological model, we have $\epsilon_G = 0$ and $\epsilon = \epsilon_c/3$.

The average density of the visible matter in the universe is only a few percent of the critical mass density. Observations of the motions of galaxies and of gravitational lensing effects, etc., show that 'dark' gravitating matter must be present. The average total density of visible plus dark matter is about 30 percent of the critical mass density. If we identify $\epsilon = \epsilon_c/3$ as the average energy density of all the matter, visible plus dark, we have quite good agreement with observation. The theory does not say anything about the nature of the dark mass.

5. Conclusions

As emphasized earlier, gravitostatics is not a new, speculative theory, but only a minimally modified form of Newtonian gravitation. In this paper, we have generalized it slightly by allowing gravitational fields to vary slowly in time, and have assumed that they satisfy a wave equation. The simplest solution of this equation implies that the average total mass density is one-third of the critical mass density, in agreement with observation. The universe has an infinite past, as measured in empirical units, and there is no horizon problem. The deceleration parameter has the value -1, which implies that the expansion rate of the universe increases with time, and which accords with observations of objects with high redshift. One does not have to postulate a mysterious 'dark energy'.

An obvious generalization of the results is given in the following Appendix. We showed above that if $\psi = -HT$ then the time-dependent term in the field equation (4.2) gives rise to a contribution to the energy density of the sources which is constant in time when measured in empirical units. If we now write the time-dependent term as $2e^{-\psi/2}{c_N}^{-1}\partial_t({c_N}^{-1}\partial_t e^{\psi/2}) = c^{-2}e^{-2\psi}B$, our result is that B is constant if $\psi = -HT$. One naturally asks what is the most general ψ that implies a constant B. We find that, in the general case, the contribution to the energy density may be less than before. The deceleration parameter may depend on the time of observation — it lies between $-1/2$ and -1 and tends to the latter value at large times.

This paper uses the conventional model of a universe with homogeneous space sections, which cannot be expected to accord exactly with obervation. The equations of any such model will differ from those found by averaging over our actual, lumpy universe, because the non-linear terms will almost certainly not average to zero. Once the distribution of dark matter is known, it should be possible to estimate the size of the error.

In summary, we have shown that a simple, rather primitive theory of gravitation can account for the large-scale structure of the universe without invoking hypotheses such as inflation or dark energy. The challenge now, for vigorous young theorists, is to find a properly relativistic theory that is at least as successful. This is not trivial because one cannot assume that the potential is a relativistic invariant (it is well known that relativistic scalar theories of gravity do not work). The examples of exact theories in Chapter IX of (Rastall 1991) do not take account of the field equation (4.2).

6. Appendix

We shall find the most general function ψ that depends only on the time coordinate t and that satisfies the equation $2e^{-\psi/2}\partial_t(e^{-3\psi/2}\partial_t\psi) = B$, where B is a constant. We write $w = e^{-3\psi/2}$ and the equation becomes $d^2w/dt^2 = -(3B/2)w^{-1/3}$. Multiplying by dw/dt and integrating gives $dw/dt = \pm(a + bw^{2/3})^{1/2}$,

where $b = -9B/2$ and a is a constant of integration. The upper sign is chosen so that $d\psi/dt < 0$. Defining $u = w^{-2/3} = e^{\psi}$, gives $t = -(3/2)\int u^{-2}(au+b)^{-1/2}du$, which is a standard integral.

The proper time T satisfies $dT/dt = e^{\psi}$. Since $dt/du = -3/2u^2S$, where $S = (ae^{\psi}+b)^{1/2}$, and $du/d\psi = u$, we have $dT/d\psi = -3/2S$, and hence $\psi' = d\psi/dT = -2S/3$, $\psi'' = (2/9)ae^{\psi}$. Eq.(2.6) implies that the redshift observed at time T_2 is

$$z = -\psi'(T_2 - T_1) + (1/2)(\psi'^2 + \psi'')(T_2 - T_1)^2, \tag{6.1}$$

provided that $T_2 - T_1$ is small, where ψ' and ψ'' are to be evaluated at T_2. The conventional form of this redshift is

$$z = H(T_2 - T_1) + (1 + q/2)H^2(T_2 - T_1)^2, \tag{6.2}$$

where H and q are the Hubble parameter and deceleration parameter at T_2. It follows that $H = 2S/3$ and $q = -1/2 - b/2S^2$. Note that q lies between $-1/2$ and -1, and that q tends to -1 when T_2 is large. If $T_2 = T_0$, the present epoch, and if the Newtonian frame is chosen so that $\psi(T_0) = 0$, then Newtonian and empirical units coincide at T_0, and $S(T_0) = (a+b)^{1/2}$. Writing $H = H_0$ and $q = q_0$, we have

$$a = (9/2)H_0(1+q_0), \qquad b = -(9/4)(1+2q_0)H_0. \tag{6.3}$$

This determines a and b in terms of the measurable H_0 and q_0. the earlier assumption that $\psi = -HT$, eq.(2.1), corresponds to $q_0 = -1$.

REFERENCES

Daly, R. A. & Djorgovski, S. G. 2003, ApJ, 597, 9
Knop, R. A., et al. 2003, ApJ, 598, 102
Misner, C. W., Thorne, K. S., & Wheeler, J. A. 1973, Gravitation (San Francisco: W. H. Freeman)
Rastall, P. 1975, Am. J. Phys., 43, 591
Rastall, P. 1991, Postprincipia: Gravitation for Physicists and Astronomers (Singapore: World Scientific)
Tonry, J. L., et al. 2003, ApJ, 594, 1